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Insensitivity of synchronization to network structure in chaotic pendulum systems with time-delay coupling

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It has been generally believed that both time delay and network structure could play a crucial role in determining collective dynamical behaviors in complex systems. In this work, we study the influence of coupling strength, time delay, and network topology on synchronization behavior in delay-coupled networks of chaotic pendulums. Interestingly, we find that the threshold value of the coupling strength for complete synchronization in such networks strongly depends on the time delay in the coupling, but appears to be insensitive to the network structure. This lack of sensitivity was numerically tested in several typical regular networks, such as different locally and globally coupled ones as well as in several complex networks, such as small-world and scale-free networks. Furthermore, we find that the emergence of a synchronous periodic state induced by time delay is of key importance for the complete synchronization. *Published by AIP Publishing.*
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The study of collective behavior of complex systems has attracted much attention of researchers in various fields due to its potential applications in physics, chemistry, biology, and engineering. We here study synchronous behavior in chaotic pendulum systems with time-delay coupling. Although it is generally believed that the topology structure of network is determinant for the dynamical behavior of complex systems, we find that the critical coupling strength for complete synchronization is the same for four network models studied here. Thus, the threshold value of coupling strength for synchronization in such networks depends strongly on the time delay of coupling, but is insensitive to the network structure in the coupled chaotic pendulum systems.

I. INTRODUCTION

Time delays are pervasive and significant in many science and application fields.[1] Many systems such as physical or biological systems interact in the form of time delay due to the fact that the interaction signal is transported with a limited speed through media. For example, in neurons, the time delays as large as 300 ms can be generated, as a result of finite speed at which action potentials propagate across neuron axons as well as time lapses occurring during both dendritic and synaptic processing.[2] The transmission delay is a sum of axonal, synaptic, and dendritic delays. The effect of a time delay on nonlinear systems has been recently studied, and various phenomena have been discovered. For example, time delay can induce oscillation death and multistability in limit-cycle oscillators.[3–8] Time delay can also be used to control cluster and synchronization in excitable Boolean networks and large laser networks.[9,10] An appropriate time delay can induce stable synchronous patterns in a network of neuronal oscillators with attractive coupling.[11]

Synchronization has been widely observed in many nonlinear systems[12,13] and attracted much attention due to its key roles in physics, biology, and sociology, etc.[14] Various phenomena have been discovered, including complete synchronization,[15–17] phase synchronization,[18,19] generalized synchronization,[20] lag synchronization.[21] Many interesting problems on synchronization have been discussed, such as analytical methods for synchronization,[22,23] applications of synchronization,[24,25] and stability conditions for synchronization,[26,27] which include the dynamical behavior of uncoupled system (periodic or chaotic), the coupling mode (active or inhibited and instantaneous or time-delayed coupling), and the topology structure of networks.

It has been found that dynamical behaviors of complex networks can be significantly influenced by its topological structure.[28–37] However, a counterexample has been found in Ref. 6 that the delay-induced oscillation death is insensitive to the complex network structure. In this work, we will mainly investigate dynamics in delay-coupled networks of chaotic pendulums. We find that the occurrence of complete

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synchronization is insensitive to the topology structure of networks in the coupled chaotic pendulums, and the critical coupling strength of synchronization is inversely proportional to the time delay.

The paper is organized as follows. In Sec. II, we will give our numerical results. Section III is devoted to theoretical analysis for regular networks. In Sec. IV, we analyze other paradigmatic types of networks to test the insensitivity. Finally, conclusions are presented in Sec. IV.

II. MODEL AND RESULTS

We consider first the neighboring-coupled chaotic pendulums[38] with time delay, described by

$$ml^2\ddot{\theta}_i + \gamma\dot{\theta}_i = -mgl\sin(\theta_i) + B + A\cos(\omega t) + \frac{\epsilon}{2d}\sum_{j=i-d,j\neq i}^{i+d}(\theta_j(t-\tau) - \theta_i(t)), \quad (1)$$

where $i = 1, 2, \ldots, N$. The parameters are set as follows and fixed throughout the paper: the mass of the oscillator m is 1, the length l is 1, the acceleration due to gravity g is 1, and the damping γ is 0.75. The d.c. torque B with $B = 0.7155$ and the a.c. torque $A\cos(\omega t)$ with $A = 0.4$ and $\omega = 0.25$ overcome the pendulum' s rotational inertia to drive the pendulum' s motion. It is found that an uncoupled pendulum is chaotic for values $l = 1.0 \pm 0.002$ with other fixed parameters.[39] It performs a libration if the length is larger than one. On the other hand, if the pendulum' s length is shorter than one, the pendulum executes a rotation. Periodic boundary conditions are chosen for the coupled pendulums, i.e., $\theta_{N+k} = \theta_k, \theta_{1-k} = \theta_{N+1-k}, k = 1, 2, \ldots, d$. Here, $2d$ $(1 \leq d \leq \frac{N}{2})$ stands for the number of coupled neighbors. So, $d = 1$ corresponds to the nearest neighboring coupled mode, and $d = \frac{N}{2}$ corresponds to the globally coupling with $N = 64$. The symbols ϵ and τ denote the coupling strength and the time delay, respectively.

To quantify the level of synchrony, we define the synchronous factor C

$$C = \frac{2}{N(N-1)}\sum_{1\leq i<j\leq N} c_{ij}, \quad c_{ij} = \frac{\int_{T_0}^{T_0+T}\theta_i\theta_j dt}{\left[\int_{T_0}^{T_0+T}\theta_i^2 dt \int_{T_0}^{T_0+T}\theta_j^2 dt\right]^{1/2}}, \quad (2)$$

where c_{ij} denotes the correlation between the i-th and j-th oscillators. Sufficiently, large T_0 and T are always selected for the transient processing and proper measurement of averaging over a long period, respectively. Clearly, we have $-1 \leq C \leq 1$ with $C = 1$ indicating complete synchronization. Figures 1(a)–1(c) illustrate the synchronous factor C as a function of coupling strength ϵ without time delay ($\tau = 0.0$) and with time delay ($\tau = 0.1, 0.2$), respectively. For $\tau = 0.0$, the complete synchronization depends on the number of neighbors $2d$, as shown by the three dashed critical lines in Fig. 1(a). In addition, the critical value of coupling strength ϵ_c deceases with increasing d with $\epsilon_c = 1.29, 0.92$, and 0.42 for $d = 2$, 6, and 10, respectively. For $\tau = 0.1$ and 0.2, however, we find that the time-delay coupling makes the system synchronous under a very weak coupling strength, i.e., $\epsilon_c \approx 0.058$ and 0.034 for $\tau = 0.1$ and 0.2, respectively. More interestingly, they are always the same for different d's, as shown in Figs. 1(b) and 1(c). Furthermore, Fig. 1(d) shows the complete synchronization occurs when time delay increases above a same critical value (e.g., $\tau_c \approx 0.06$ for $\epsilon = 0.1$) for different coupling structures ($d = 10$, 6, and 2).

To demonstrate the dependence of the critical coupling strength ϵ_c on d, Fig. 2(a) summarizes the relation between ϵ_c and d. A monotonic decrease of ϵ_c on d is observed in the absence of time delay, i.e., at $\tau = 0$. But differently, a constant $\epsilon_c \approx 0.058$ (or 0.034) is found for an arbitrary d with a

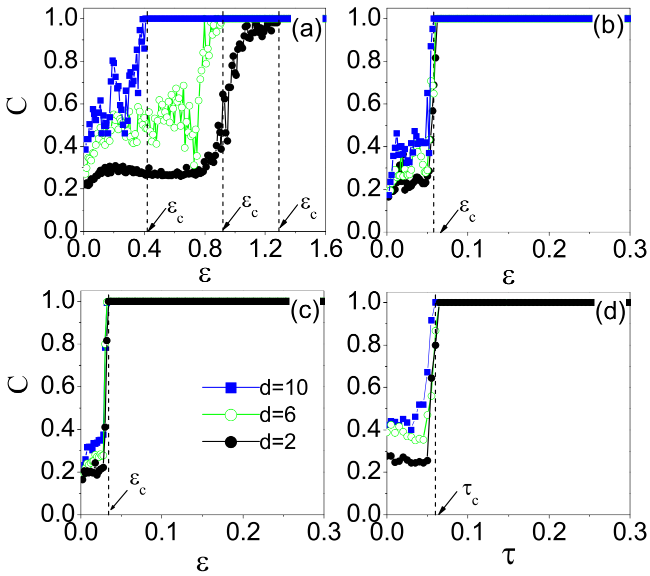

FIG. 1. Plots of synchronous factor C against ϵ for $\tau = 0$ (a), $\tau = 0.1$ (b), and $\tau = 0.2$ (c), respectively. In (a), $\epsilon_c \approx 0.42$, 0.92, and 1.29 for $d = 10$, 6, and 2, respectively. In (b), $\epsilon_c \approx 0.058$ and in (c) $\epsilon_c \approx 0.034$ which are unchanged for different d's. (d) The synchronous factor C versus τ for $\epsilon = 0.1$. Here, three different coupling structures are considered with $d = 10$ (square), 6 (open circle), and 2 (solid circle).

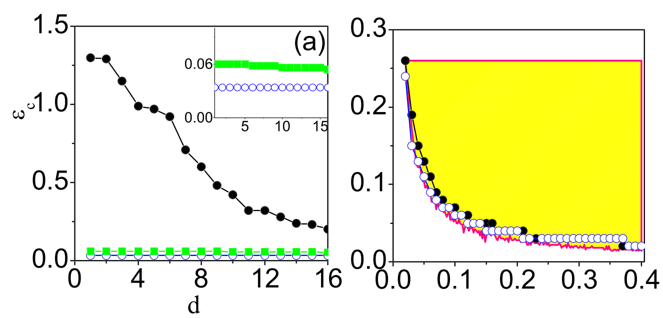

FIG. 2. (a) Plots of critical values of coupling strength as a function of d with $\tau = 0.0$ (solid circles), $\tau = 0.1$ (hollow circles), and 0.2 (solid squares). The inset of (a) as a zoomed-in part of (a) shows the identical value of $\epsilon \approx 0.058$ and 0.034 for $\tau = 0.1$ and 0.2, respectively. (b) Region of complete synchronization of the delay-coupled system of Eq. (1) on the $\epsilon - \tau$ space for $d = 2$ (solid circles) and 10 (hollow circles). The critical curve for complete synchronization is determined by $C = 1$. The pink solid line corresponds to the critical curve for the transition from chaotic to periodic states. These three curves nearly overlap with each other. The pink solid curve is determined by $\Lambda' = 0$, in which Λ' is the transverse Lyapunov exponent of the synchronous manifold.

fixed time delay, $\tau = 0.1$ (or 0.2), giving an insensitive dependence on d. To further demonstrate such an independence, the phase diagram of synchronization on the $\epsilon - \tau$ plane is shown in Fig. 2(b) with $N = 64$ and $d = 2$, 10. From Fig. 2(b), we clearly see that the synchronous and asynchronous regions are divided by the critical curves, and those critical curves are nearly the same for different d's. As a result, the insensitivity of the critical coupling strength to the number of connections d holds for different τ's.

To understand how the time delay influences the collective dynamical behavior, we discuss the bifurcation of the average angular velocity which is defined by

$$\Theta(jT') = \frac{1}{N}\sum_{i=1}^{N}\dot{\theta}_i(jT'), \tag{3}$$

at times that are integer multiples of the forcing period $T' = 2\pi/\omega$. Figure 3(a) gives the average angular velocity at $t = 60T', 61T', \ldots, 80T'$ with $\tau = 0.2, N = 64$ for $d = 2$. The chaotic states (which can be tested by time series) are preserved for small ϵ but they become periodic for larger ϵ as it comes across a critical value of $\epsilon_c \approx 0.034$. Similarly, the case for $d = 10$ is shown in Fig. 3(b). Figure 3 indicates that the critical couplings for the system transition from chaotic to periodic states are the same for different d's. As a fact, this value of $\epsilon_c = 0.034$ equals to the critical couplings for the system transition from asynchronous to synchronous states found in Fig. 1(c). Therefore, we suggest that time-delay coupling induces a transition from chaotic to periodic, resulting in the occurrence of complete synchronization.

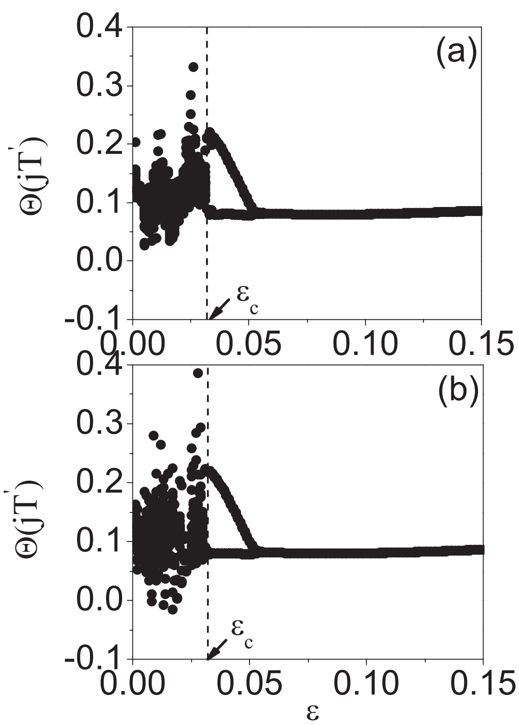

FIG. 3. Average velocities Θ plotted against ϵ, with Θ taken at $t = 60T', 61T', \ldots, 80T'$ ($T' = 2\pi/\omega$) for each ϵ at $d = 2$ (a) and $d = 10$ (b), respectively. Here, $\tau = 0.2$ and $N = 64$. An identical critical value ϵ_c for system transition from chaotic to periodic states exists at $\epsilon_c \approx 0.034$.

III. ANALYTIC SOLUTION

In this section, we consider a theoretical analysis on the mechanism of synchronization induced by time-delay coupling. For the stability of complete synchronization of coupled chaotic systems, we follow the method of master stability functions for local stability of complete synchronization.[40–42] Then, Eq. (1) becomes

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i) + \frac{\epsilon}{k_i}\sum_{j=1}^{N} g_{i,j}\Gamma(\boldsymbol{x}_j(t-\tau) - \boldsymbol{x}_i(t)), \tag{4}$$

where $\boldsymbol{x}_i \in R^n$ is a dynamical variable vector and the function $f = [f_1, f_2, \ldots, f_n]^T$ is nonlinear, in general. Γ denotes a $n \times n$ constant matrix linking coupled variables. If the nodes j and i are connected by a link, $g_{i,j} = 1$, and otherwise $g_{i,j} = 0.0$, $g_{i,i} = 0.0$; namely, self-connection is not allowed. $k_i = \sum_{j=1}^{N} g_{i,j}$ is degree of the node i. According to Eq. (4), Eq. (1) can be written as follow:

$$\dot{x_i^1} = x_i^2, \tag{5a}$$

$$\dot{x_i^2} = \frac{1}{ml^2}\Bigg[-\gamma x_i^2 - mgl\sin x_i^1 + B + A\cos(\omega t) + \frac{\epsilon}{k_i}\sum_{j=1}^{N} g_{i,j}(x_i^1(t-\tau) - x_i^1(t))\Bigg], \tag{5b}$$

where $x_i^1 = \theta_i$ and $x_i^2 = \dot{\theta}_i$.

The synchronization state resides on the synchronous manifold defined by $M = \{\boldsymbol{x}_1 = \boldsymbol{x}_2 = \ldots = \boldsymbol{s}(t)\}$, and according to $k_i = \sum_{j=1}^{N} g_{i,j}$, its solution satisfies the following form:

$$\dot{\boldsymbol{s}} = \boldsymbol{f}(\boldsymbol{s}) + \epsilon\Gamma(\boldsymbol{s}(t-\tau) - \boldsymbol{s}(t)). \tag{6}$$

Clearly, now the solution depends on ϵ also. Note that this is fundamentally different with the synchronization state in the absence of time delay ($\tau = 0$), where the coupling term will naturally vanish.[40–42]

The stability of the synchronization state can be analyzed by setting $\boldsymbol{x_i}(t) = \boldsymbol{s}(t) + \boldsymbol{\eta_i}(t)$, where $\boldsymbol{\eta_i}(t)$ represents a very small deviation of j-th oscillator from the synchronous manifold $\boldsymbol{s}(t)$, and inserting it into Eq. (4), the linearization equations can be obtained as follows:

$$\dot{\boldsymbol{\eta}}_i = D\boldsymbol{f}(\boldsymbol{s})\boldsymbol{\eta_i}(t) + \frac{\epsilon}{k_i}\sum_{j=1}^{N} g_{i,j}\Gamma(\boldsymbol{\eta}_j(t-\tau) - \boldsymbol{\eta}_i(t)), \tag{7}$$

where $D\boldsymbol{f}(\boldsymbol{s})$ is a Jacobi matrix of $\boldsymbol{f}$ on the synchronous manifold $\boldsymbol{s}$. Denote $\boldsymbol{\eta}(t) = (\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_N)^T$, they can be rewritten in a compact form

$$\dot{\boldsymbol{\eta}}(t) = [I_N \otimes (D\boldsymbol{f}(\boldsymbol{s}) - \epsilon\Gamma)]\boldsymbol{\eta}(t) + \epsilon G \otimes \Gamma\boldsymbol{\eta}(t-\tau), \tag{8}$$

where $\otimes$ represents the Kronecker product, I_N denotes the N-dimensional identity matrix, and $G = \left\{\frac{g_{i,j}}{k_i}\right\}_{N\times N}$ is a coupling matrix which can be diagonalized with a matrix A, namely,

$$AGA^{-1} = diag(\lambda_0, \lambda_1, ..., \lambda_{N-1}), \tag{9}$$

where λ_i's are the eigenvalues of the matrix G, ordered by

$$1.0 = \lambda_0 \geq \lambda_1 \geq ... \geq \lambda_{N-1} \geq -1.0. \tag{10}$$

In particular, for the neighboring-coupling, the eigenvalues are

$$\lambda_k = \frac{1}{d}\sum_{j=1}^{d} \cos\frac{2jk\pi}{N}, \quad k = 0, 1, 2, ..., N-1. \tag{11}$$

By setting $\boldsymbol{\delta}(t) = A\boldsymbol{\eta}(t)$, Eq. (8) becomes

$$\dot{\boldsymbol{\delta}}(t) = [I_N \otimes (D\boldsymbol{f}(\boldsymbol{s}) - \epsilon\Gamma)]\boldsymbol{\delta}(t) + \epsilon AGA^{-1} \otimes \Gamma\boldsymbol{\delta}(t-\tau). \tag{12}$$

Then, we can transform Eq. (8) into the following equations for the N independent modes:

$$\dot{\boldsymbol{\delta}}_k(t) = (D\boldsymbol{f}(\boldsymbol{s}) - \epsilon\Gamma)\boldsymbol{\delta}_k(t) + \epsilon\lambda_k\Gamma\boldsymbol{\delta}_k(t-\tau). \tag{13}$$

The $k = 0$ mode describes the dynamical behavior of the synchronous manifold [Eq. (6)]. The other modes for $k \neq 0$ govern the transverse stability of the synchronous state. For Eq. (1), the equation of perturbation can be obtained:

$$\dot{\delta}_k^1 = \delta_k^2, \tag{14a}$$

$$\dot{\delta}_k^2 = \frac{1}{ml^2}\left[-\gamma\delta_k^2 - mgl\cos s^1\delta_k^1\right] + \epsilon(\lambda_k\delta_k^1(t-\tau) - \delta_k^1(t)), \tag{14b}$$

where $s^1 = \theta$ in Eq. (17). We can define the transverse Lyapunov exponent of complete synchronization as follows:[43]

$$\Lambda_k = \lim_{T\to+\infty}\frac{1}{T}\ln\frac{\left\{\int_{-\tau}^{0}||\boldsymbol{\delta}_k(T+t)||dt\right\}^{1/2}}{\left\{\int_{-\tau}^{0}||\boldsymbol{\delta}_k(t)dt||\right\}^{1/2}}, \tag{15}$$

where $||\cdot|| = \sqrt{(\delta_k^1)^2 + (\delta_k^2)^2 + \cdots + (\delta_k^n)^2}$ is the modulus of the vector $\boldsymbol{\delta}_k$. The largest transverse Lyapunov exponent $\Lambda = \max(\Lambda_0, \Lambda_1, ..., \Lambda_{N-1})$ which governs the stability of the synchronization state can be calculated. In particular, if $\Lambda < 0$, we will be able to observe a stable complete synchronization.

Figures 4(a) and 4(b) show the largest transverse Lyapunov exponent Λ of the neighboring-coupled chaotic pendulums in the $\epsilon - d$ parameter space for $\tau = 0.1$ and 0.2, respectively. From Fig. 4, we see that similarly the parameters plane is divided into green ($\Lambda > 0$) and pink ($\Lambda < 0$) parts by the vertical dashed line. Roughly the largest transverse Lyapunov exponent Λ becomes negative for all d if the coupling strength is above the identical critical value ϵ_c ($\epsilon_c \approx 0.058$ and 0.034 for $\tau = 0.1$ and 0.2, respectively). It indicates that the coupled chaotic pendulums with different neighbors d have an identical critical coupling strength for synchronization. Thus, the critical coupling strength of synchronization is insensitive to the number of connection d.

In addition, we find that the critical value ϵ_c for the transverse stability of the synchronous state [e.g., $\epsilon_c = 0.034$ for $\tau = 0.2$ in Fig. 4(b)] is roughly equal to the critical coupling for system transition from chaotic to periodic states in Fig. 3 where $\epsilon_c = 0.034$ for $\tau = 0.2$ again. This point is understandable based on the similar linearization form for the stability analysis on the synchronous manifold:

$$\dot{\boldsymbol{\delta}}'(t) = (D\boldsymbol{f}(\boldsymbol{s}) - \epsilon\Gamma)\boldsymbol{\delta}'(t) + \epsilon\Gamma\boldsymbol{\delta}'(t-\tau), \tag{16}$$

where $\boldsymbol{\delta}'(t) = \boldsymbol{\delta}_0(t)$ for $\lambda_0 = 1.0$. The largest Lyapunov exponent Λ', which characterizes the dynamics of synchronous manifold, can be computed according to Eq. (15). $\Lambda' > 0$ and $\Lambda' = 0$ correspond to the chaotic and periodic state, respectively. The critical curve for periodic state which is determined by $\Lambda' = 0$ is shown in Fig. 2(b). Roughly, the critical curve for periodic state shows a good agreement with these critical curves of complete synchronization. Therefore, we believe that the mechanism of synchronization of delayed system is due to time-delay coupling induced transition from chaotic to periodic states.

Furthermore, for the chaotic pendulums in Eq. (1), the equation of synchronous manifold can be specially expressed as

$$ml^2\ddot{\theta} + \gamma\dot{\theta} = -mgl\sin(\theta) + B + A\cos(\omega t) + \epsilon(\theta(t-\tau) - \theta(t)). \tag{17}$$

Based on the approximation of $\theta(t-\tau) = \theta(t) - \tau\dot{\theta}(t)$ for $\tau \ll 1$, we can estimate its synchronization threshold precisely for a fixed time delay from the dynamical equation of

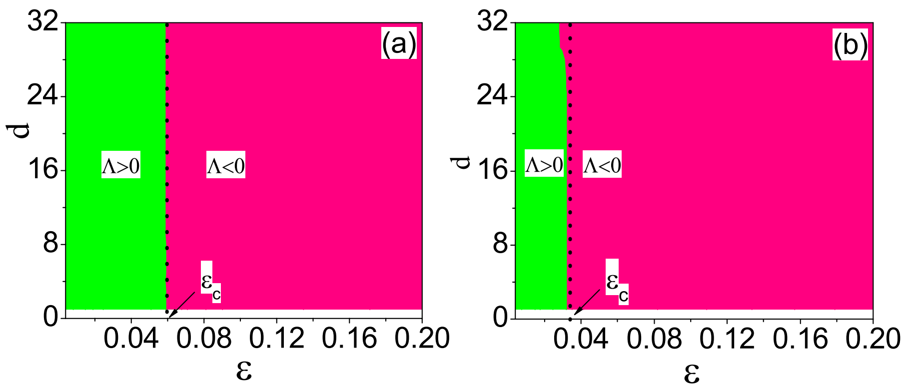

FIG. 4. The largest transverse Lyapunov exponent Λ on the $\epsilon - d$ plane for $\tau = 0.1$ (left panel) and 0.2 (right panel), respectively. Two planes are divided into two parts ($\Lambda > 0$ and $\Lambda < 0$) by the critical coupling strength ϵ_c at 0.058 (for left panel) and 0.034 (for right panel), respectively, showing that Λ highly depends on ϵ, but not on d.

synchronous manifold [Eq. (17)], which is now reduced to the following equation:

$$ml^2\ddot{\theta} + \gamma\dot{\theta} = -mgl\sin(\theta) + B + A\cos(\omega t) + \epsilon(\theta(t) - \tau\dot{\theta(t)} - \theta(t)), \quad (18)$$

and further,

$$ml^2\ddot{\theta} + (\gamma + \epsilon\tau)\dot{\theta} = -mgl\sin(\theta) + B + A\cos(\omega t). \quad (19)$$

Now Eq. (19) becomes uncoupled, showing the same equation form with Eq. (1) for the uncoupled pendulums. The coupling strength and time delay have come into the evolution equation with the redefined damping coefficient, and the combinative action of time delay and coupling strength will be important for dynamical behavior of system.

In Fig. 5(a), we plot the bifurcation diagram of the average angular velocity with the change of γ. A critical γ_c for a transition from chaotic to periodic states exists, i.e., $\gamma_c \approx 0.757$. Thus, the system can transfer from a chaotic state to a periodic one if $\gamma + \epsilon\tau > \gamma_c$. Thus, we can deduce a precise estimation for the critical coupling strength

$$\epsilon_c = \frac{\gamma_c - \gamma}{\tau}. \quad (20)$$

As both $\gamma_c = 0.757$ and $\gamma = 0.75$ (chosen as a system parameter in the paper and fixed) are constant, ϵ_c is then in inverse proportion with τ. As given in Fig. 5(b), this analytical result (blue dashed line) shows a good agreement with the numerical results from Eq. (17) (pink solid line). A deviation is clear for larger τ, as our approximative method highly relies on the condition of a small time delay ($\tau \ll 1$).

IV. OTHER TYPES OF NETWORKS

So far we have shown the insensitive dependence of synchronization on the regularly connected ring structures with different coupling distances d in the delay-coupled chaotic pendulums. Next, we show that such an independence can be easily applied to other complex networks. As paradigmatic examples, the globally coupling network, ring network with nearest neighbor coupling but zero-flux boundary, star network, and 8×8 grid network are computed with $N = 64$ and $\tau = 0.1$, several complex networks, including small-world networks for different connecting probability p's ($p = 0.1$, 0.5, and 0.9) and scale-free networks with different average degree $\langle k \rangle$'s ($\langle k \rangle = 2$ and 4) are tested with $N = 200$ and $\tau = 0.2$. Here, we apply the standard construction algorithms for the small-world networks which are adding links with random reconnection probability p and that for scale-free networks which is constructed by preferential attachment. We find that the critical value $\epsilon_c \approx 0.058$ and 0.034 are always unchanged for $\tau = 0.1$ and 0.2, as illustrated in Figs. 6(a) and 6(b) and Figs. 6(c) and 6(d), respectively.

Finally, it remains of great interest to verify our main results for other chaotic systems. As one example, Fig. 6(f) shows the critical coupling strength ϵ_c as a function of d in the neighboring coupled chaotic Rossler system, described by

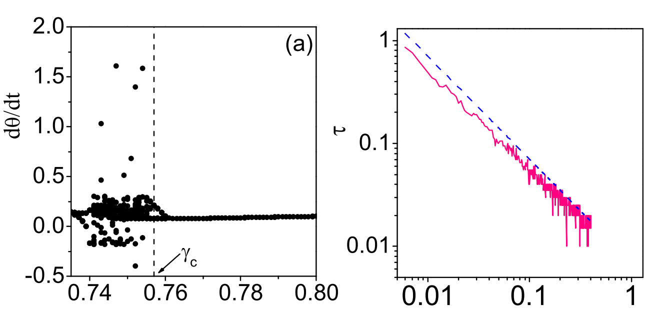

FIG. 5. (a) Bifurcation diagram of the average angular velocity against γ. A critical $\gamma_c \approx 0.757$ is obtained to show the transition from chaotic to periodic states. (b) Plots of τ against ϵ_c. The blue dashed line comes from the analysis of Eq. (20), and the pink solid line which is determined by $\Lambda' = 0$ stands for the numerical results from Eq. (17).

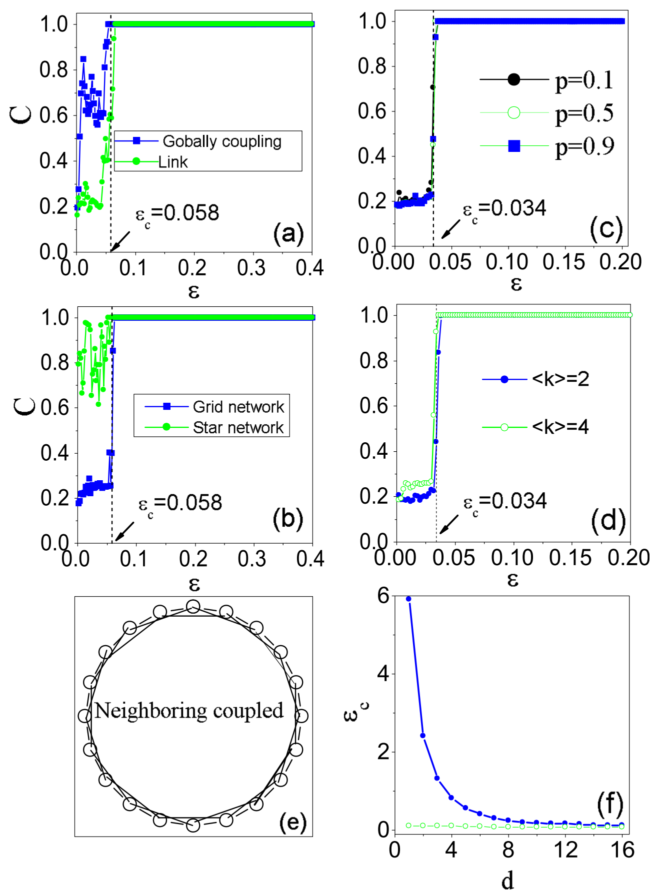

FIG. 6. Demonstration of insensitive dependence of time-delay coupling induced synchronization on different types of complex networks. (a) and (b) The synchronous factor C versus ϵ for globally coupling network, the link network with nearest neighbor coupling but zero-flux boundary conditions (a), star network and 8×8 grid network (b) with $N = 64$ and $\tau = 0.1$. (c) and (d) Plots of the corresponding synchronous factor C against ϵ for the small-world network (c) with different p's ($p = 0.1$, 0.5, and 0.9) and the scale-free network (d) with different average degree $\langle k \rangle$'s ($\langle k \rangle = 2$ and 4) for $N = 200$ and $\tau = 0.2$, respectively. It shows that $\epsilon_c \approx 0.058$ and 0.034 are unchanged for $\tau = 0.1$ and 0.2. (e) Schematic diagrams of regular ring network with neighbor coupling. (f) Plot of critical coupling strength ϵ_c in regular ring network of coupled chaotic Rossler system as a function of d with $\tau = 0.0$ (solid circles) and $\tau = 1.0$ (hollow circles).

$$\dot{x}_i = -y_i - z_i, \tag{21a}$$

$$\dot{y}_i = x_i + 0.15 y_i + \frac{\epsilon}{2d} \sum_{j=i-d, j\neq i}^{i+d} (y_j(t-\tau) - y_i(t)), \tag{21b}$$

$$\dot{z}_i = 0.4 + z_i(x_i - 8.5). \tag{21c}$$

Again it proves that the critical values of coupling strength greatly decrease in the absence of time delay and are not insensitive to the number of coupled neighbor with the time delay.

V. CONCLUSION AND DISCUSSION

In summary, we have studied complete synchronization in the delay-coupled chaotic pendulums. Extensive numerical results, supplemented by a theoretical analysis from the stability analysis of synchronous states, reveal that the delay-coupled chaotic pendulums can transfer from an asynchronous state to a synchronous one under a nearly identical coupling strength, accompanying with the system transition from chaotic to periodic states. The synchronization threshold is determined by the time delay of coupling and independent on how the chaotic oscillators are coupled. Furthermore, we have demonstrated both numerically and theoretically that the underlying mechanism is the time delay induced transition from chaotic to periodic states. This insensitivity to network structure has been tested with several typical regular (such as different locally coupled networks and globally coupled network) and complex networks (such as small-world network and scale-free network), and the similar qualitative results can be found in delay-coupled Rossler chaotic system with special parameters. Below it is necessary to give some further discussions, and it is interesting to test the insensitivity on other chaotic systems. Finally, we investigate the neighboring coupled chaotic Lorenz systems with two sets of special parameters, described by

$$\dot{x}_i = \sigma(y_i - x_i) + \frac{\epsilon}{2d} \sum_{j=i-d, j\neq i}^{i+d} (y_j(t-\tau) - y_i(t)), \tag{22a}$$

$$\dot{y}_i = 28.0 x_i - y_i - x_i z_i, \tag{22b}$$

$$\dot{z}_i = x_i y_i - z_i, \tag{22c}$$

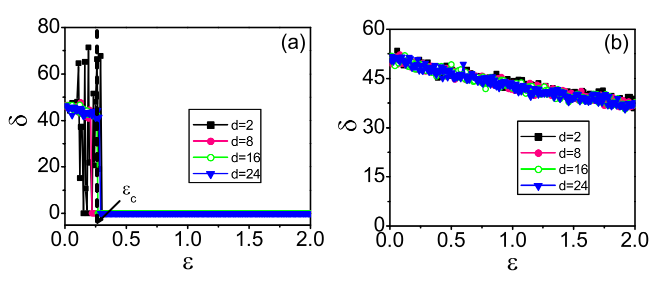

FIG. 7. (a) and (b) Plots of synchronous factor δ against ϵ for $\tau = 0.1$ and $N = 64$ with $\sigma = 2.44$ (a) and $\sigma = 6.0$ (b), respectively. In (b), $\epsilon_c \approx 0.26$. Here, four different coupling structures are considered with $d = 2$ (square), 8 (solid circle), 16 (open circle), and 24 (triangle).

where $i = 1, \ldots, N$. $N = 64$. Figures 7(a) and 7(b) show the synchronous factor $\delta = \langle \frac{1}{N-1} \sum_{j=2}^{N} (x_i - x_1)^2 \rangle$ with $\tau = 0.1$ for $\sigma = 2.44$ and 6.0, respectively, where $\langle \cdot \rangle$ is time average. $\delta = 0$ indicates complete synchronization. From Fig. 7(a), we find that the insensitivity to the number of connection d can be observed in the neighboring coupled chaotic Lorenz systems with $\sigma = 2.44$, as the critical values of coupling strength are approximately the same for different d's. However, the complete synchronization does not occur for $\sigma = 6.0$, and this insensitivity cannot be observed for $\sigma = 6.0$ [Fig. 7(b)]. It indicates that this insensitivity to network structure really relies on the system or the parameter of system. Although we have tested many numerical simulations for other chaotic systems, it is not easy to find this insensitivity, since the time delays not only affect the dynamical behavior of the synchronous manifold [Eq. (6)] but also influence the stability of synchronous manifold [Eq. (12)]. Since in many physical and biological systems, time delay are unavoidable, we believe that the findings in our paper can be extended to other realistic coupled systems and to provide further insight into information processing.

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