A secure communication scheme based on the phase synchronization of chaotic systems

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Phase synchronization of chaotic systems with both weak and strong couplings has recently been investigated extensively. Similar to complete synchronization, this type of synchronization can also be applied in secure communications. We develop a digital secure communication scheme that utilizes the instantaneous phase as the signal transmitted from the drive to the response subsystems. Simulation results show that the scheme is difficult to be broken by some traditional attacks. Moreover, it operates with a weak positive conditional Lyapunov exponent in the response subsystem. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1564934]

Recently, secure communication schemes utilizing the synchronization of chaos have mainly been based on the complete synchronization between the drive and the response chaotic subsystems. All the schemes require that the response subsystem should possess negative conditional Lyapunov exponents. In this article, we propose a secure communication scheme based on the phase synchronization in the drive-response chaotic system. Various characteristics of the scheme are studied. The proposed scheme not only extends the potential applications of phase synchronization, but also leads to an alternative way to design secure communication. Moreover, the different characteristics of complete and phase synchronization benefit the development of secure communication with chaotic systems.

I. INTRODUCTION

An interesting phenomenon of practical importance in coupled chaotic oscillators is the synchronizing state. Various types of synchronization including *complete synchronization* (CS),[1–3] *generalized synchronization*,[4–8] and *phase synchronization* (PS)[9–13] have been studied. Among them, CS has been considered as a candidate for secure communications. Two typical approaches for this purpose include the masking of a weak analog message signal onto a strong chaotic signal from drive to response subsystems[14–16] and the modulation of the parameters of the drive subsystem by a digital message.[17,18] Besides these approaches, there are also other more complicated methods. They include the synchronization of hyperchaotic systems whose geometric structures are more complex;[19–22] the use of volume-preserving maps,[23] which do not possess an attractor and have essentially space-filling trajectories; the synchronization through impulsive

coupling,[24–27] the method based on the active–passive decomposition with randomly multiplexed scalar coupling,[28–30] and information masked by chaotic signal of a time-delay system.[31,32] In order to improve the noise tolerance capability, an encoding method with a lower transmission rate can be considered, which by using we can encode a binary symbol in N continuous rotations of the mean attractor.[33,34] Most of the above-mentioned approaches are based on CS that requires negative conditional Lyapunov exponents (CLEs) of the response subsystem.[35,36] This condition is important in maintaining the robust CS between the drive and the response subsystems during the communication.

On the other hand, PS is distinguished from CS by the appearance of entrainment between the phases of interacting systems with little correlation in signal amplitudes.[37,38] It has been discovered to be a key feature in the dynamics of the human cardiorespiratory system,[39] extended ecological system,[40] magnetoencephalographic activity of Parkinsonian patients,[41] and electrosensitive cells of paddlefish.[42] In general, if the phase difference θ of two chaotic oscillators has a bounded value, i.e., $|\theta| <$ const, it can be considered as PS.[9–11]

In this paper, we present a scheme that hides binary messages in the instantaneous phase of the drive subsystem used as the transmitting signal to drive the response subsystem. At the response subsystem, the phase difference is detected and its strong fluctuation above or below zero recovers the transmitted binary message at certain coupling strength. This scheme is difficult to be broken by some traditional attacks, and it also operates with a weak positive CLE in the response subsystem.

The layout of the paper is as follows. The proposed secure communication scheme and its dynamical mechanism are described in Sec. II. Based on the scheme, we further restrict the transmitted signal to have bounded amplitudes instead of a linear phase increment. Thus the signal can be transmitted through physical channels. In Sec. III, an easy
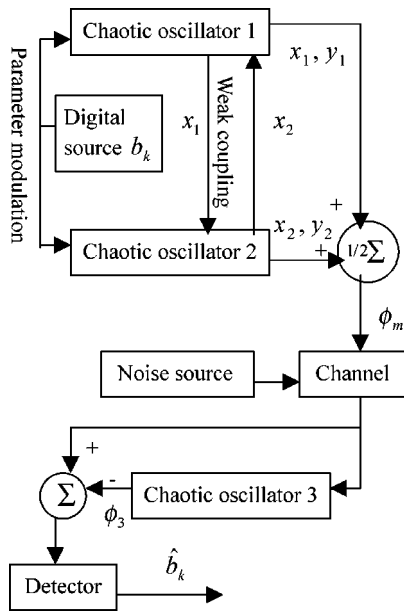
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FIG. 1. Secure communication scheme with instantaneous phase as the transmitted signal.

FIG. 2. Projections of the phase portraits: (a) Rössler oscillators (x_1,y_1), and (b) mean fields (x_m,y_m) with $\Delta\omega=0$. The straight line in (b) corresponds to Poincaré surface. Equations (1) and (2) are numerically solved using the fourth-order Runge–Kutta method with time step 1×10^{-3}.

method is also proposed to detect the information bit from the phase difference between the transmitted signal and the response phase. In Sec. IV, the robustness of communication is discussed. Simulations show that the scheme has certain tolerance on noise and parameter mismatch. Section V shows briefly that it is difficult to break the scheme with some traditional attacks. Finally, conclusions are drawn in the last section.

II. SECURE COMMUNICATION SCHEME

A secure communication scheme based on the phase signal coupling approach proposed in our previous paper[43] is developed. We have shown that at strong coupling, robust PS can be obtained in a drive-response system using phase information as the transmitted signal. Here, we utilize this model to obtain secure communication by means of PS. A block diagram of the proposed scheme is shown in Fig. 1. Suppose that there are three identical chaotic oscillators (x_i,y_i,z_i) with $i=1,2,3$. In the drive subsystem, two identical chaotic oscillators (1 and 2) are coupled with state variables x_1 and x_2. The coupling is typical and PS can be obtained at weak coupling.[9–11,44] Furthermore, we use the mean value of two corresponding variables to obtain the instantaneous phase and utilize it as the transmitted signal. This means that the transmitted information ϕ_m is the phase of mean field (x_m,y_m) with $x_m=(x_1+x_2)/2$ and $y_m=(y_1+y_2)/2$. At the receiver, ϕ_m drives chaotic oscillator 3 using phase coupling method at strong coupling.[43] The phase synchronization between ϕ_m and ϕ_3 can also be obtained with small fluctuations. If a binary sequence b_k is used to modulate the same parameter in both oscillators 1 and 2, the information can be hidden in ϕ_m and transmitted to the response subsystem through a communication channel. With certain coupling, the corresponding binary information $\hat{b}_k$ can be recovered from the fluctuation of phase difference
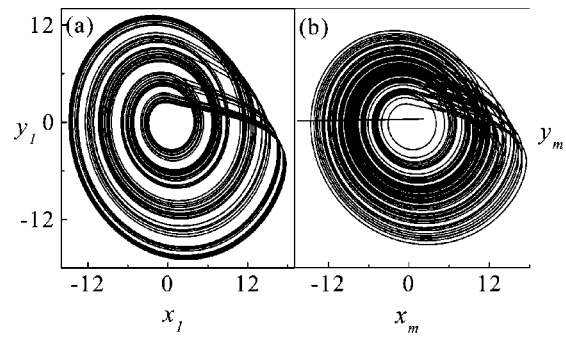
between ϕ_m and ϕ_3. When an additive noise source is added to the channel, the system can allow certain tolerance on noise.

In the following, coupled Rössler oscillators are utilized as an example to illustrate the secure communication process. The equations for the drive subsystem are[45]

$$\dot{x}_{1,2}=-(\omega+\Delta\omega)y_{1,2}-z_{1,2}+\varepsilon(x_{2,1}-x_{1,2}),$$
$$\dot{y}_{1,2}=(\omega+\Delta\omega)x_{1,2}+0.15y_{1,2},\qquad(1)$$
$$\dot{z}_{1,2}=0.2+z_{1,2}(x_{1,2}-10).$$

The response subsystem is governed by

$$\dot{x}_3=-\omega'y_3-z_3+\eta(r_3\cos(\phi_m)-x_3),$$
$$\dot{y}_3=\omega'x_3+\alpha y_3,\qquad(2)$$
$$\dot{z}_3=0.2+z_3(x_3-10).$$

Here parameters $\omega=\omega'=1$, ε, and η are the coupling strengths in the drive and the response subsystem, respectively. If not specified, we always set $\alpha=0.15$, $\varepsilon=5\times10^{-3}$, and $\eta=5.3$ throughout this paper. The parameter mismatch $\Delta\omega$ is modulated by a binary message. For practical applications, it is not difficult to design the Rössler oscillators with an analog computer.[46]

Amplitude of the response subsystem is defined as

$$r_3=(x_3^2+y_3^2)^{1/2}.\qquad(3)$$

At the transition to PS, ε is almost equal to $\Delta\omega$.[44] As the two oscillators are identical, even a small ε can make them achieve PS. The phase portraits of attractors (x_1,y_1) and (x_m,y_m) after PS are shown in Figs. 2(a) and 2(b). They have similar geometric structures. As the Rössler attractors and the mean field shown in Figs. 2(a) and 2(b) have single rotation center, their instantaneous phase can be simply obtained by

$$\phi_j=\arctan\left(\frac{y_j}{x_j}\right)\quad\text{with }j=1,2,3,m.\qquad(4)$$

In the scheme, we use ϕ_m as the transmitted signal instead of ϕ_1 (or ϕ_2) to enhance the security. This is due to the fact that the trajectory of (x_m,y_m) has a more complex return map of rotation period [as shown in Fig. 7(b) that will be
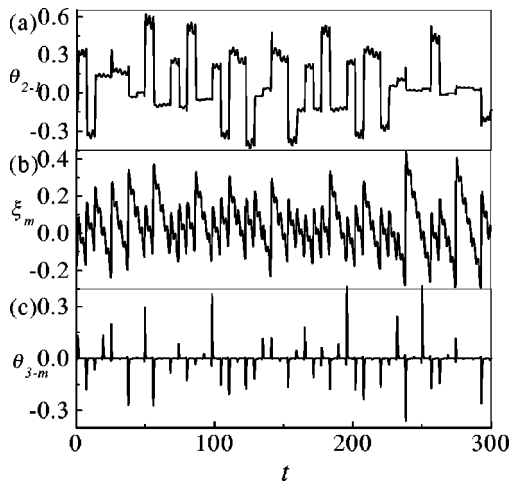
FIG. 3. Time evolution of (a) phase difference θ_{2-1} in the drive subsystem, (b) phase fluctuation of mean field ξ_m with mean frequency $\Omega_m = 1.034$, and (c) phase difference θ_{3-m} in the response subsystem. Here we set $\Delta\omega = 0$.

FIG. 4. Time evolution of (a) the binary sequence b_k; (b) the phase fluctuation of the transmitted signal ξ_m; (c) and (d) the phase difference θ_{m-3} with $\theta_{m-3} = \phi_m - \phi_3$; (e) detected binary information $\hat{b}_k$.

discussed later]. If ϕ_m is considered on the real line, the rate of this increase will typically vary in a chaotic manner.[47] This means that the rate of increase of the phase variable can be modeled as a mean steady drift with a zero mean chaotic fluctuation. Thus, we can write

$$\phi_m = \Omega_m t + \xi_m, \tag{5}$$

where Ω_m is the average frequency. The term ξ_m can be interpreted as chaotic fluctuation[47] or effective phase noise[37,38] as shown in Fig. 3(b). Figure 3(a) shows the PS between ϕ_1 and ϕ_2 at weak coupling where $\theta_{2-1} = \phi_2 - \phi_1$ while Fig. 3(c) is the PS between ϕ_m and ϕ_3 at strong coupling. Both phase differences have amplitude fluctuations.

To modulate the information on the instantaneous phase, we let $\Delta\omega$ change with the binary information in the following way:

$$\Delta\omega = \begin{cases} 0.01 & \text{if bit to be transmitted} = 1 \\ -0.01 & \text{if bit to be transmitted} = 0. \end{cases} \tag{6}$$

Each time when y_m passes through zero in the positive direction which is shown as the Poincaré surface in Fig. 2(b), $\Delta\omega$ is triggered to the value corresponding to the next binary information. Therefore, the time period for one bit is just the duration for one rotation. If we transmit a sequence of bits plotted in Fig. 4(a), the corresponding ξ_m still appears as random fluctuation, as observed in Fig. 4(b). At the response subsystem, the phase difference θ_{m-3} shows a PS state with small amplitude fluctuations, as observed in Fig. 4(c). From this figure, it seems difficult to extract binary information from the fluctuation. However, if we only observe the fluctuation between 0.01 and -0.01 as plotted in Fig. 4(d), the binary information can be found from the different appearances of the amplitude. When the amplitude fluctuates above zero at most of the time, it represents the bit "1." Otherwise, it is "0." By this means, the corresponding binary information $\hat{b}_k$ can be detected successfully, as shown in Fig. 4(e).

The communication mechanism can be explained as follows: Before the information bit is modulated onto $\Delta\omega$, the
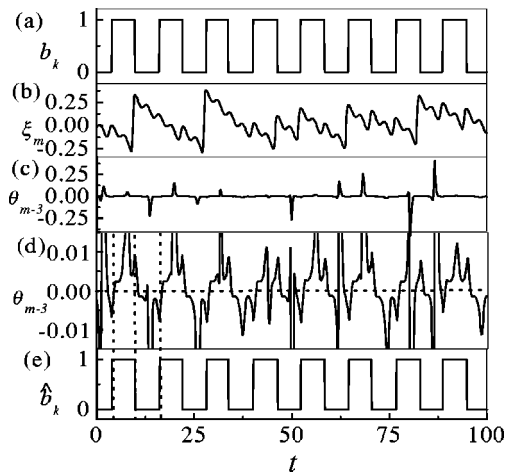
two coupled and identical oscillators in Eq. (1) approach PS at a small coupling strength. This is considered as PS under weak coupling.[44] In particular, ϕ_m is the phase of their mean fields and can be considered as $\phi_m \approx \phi_{1,2}$, as observed in Fig. 3(a). However, the coupling is a strong one for the case of the response subsystem,[43] with $\phi_3 \cong \phi_m$ but their difference has occasional large fluctuations over small ones. If we neglect the intermittent large fluctuations shown in Fig. 3(c), similar chaotic fluctuations are produced in most cases at strong coupling strength, i.e., $\xi_3 \approx \xi_m$. For Rössler oscillators, the average frequency is almost equal to ω, i.e., $\Omega_{1,2,m} \approx \omega$. After the information bit is modulated onto $\Delta\omega$, the two chaotic oscillators in Eq. (1) are still identical and so $\phi_m \approx \phi_{1,2}$ is maintained. On the other hand, as the first two functions (i.e., x and y functions) in Eq. (1) both have the same ω and $\Delta\omega$, the result is that the average frequencies in Eq. (5) could be written as $\Omega_{1,2,m} \approx \omega + \Delta\omega$. By this means, the information bits are modulated into the average frequency of the instantaneous phase. If this frequency varies much smaller than the amplitude of chaotic fluctuations ξ_m, the drive signal with the hidden information bits still appears noise like. In our example, the change of mean frequency for modulating "1" or "0" is $\Delta\omega$, which is rather small. The noisy phenomena are shown in Fig. 4(b). By this means, the information bits can be extracted only if the exact trajectory of ξ_m is known. Because $\Delta\omega$ mainly affects Ω_m, $\xi_3 \approx \xi_m$ is maintained at strong coupling when the information bits are utilized for modulation. The feedback term $\eta(\cdot)$ in Eq. (2) also makes their average frequency close to each other at all time, i.e., $\Omega_3 = \Omega_m$. Thus, $\eta(\cdot) \neq 0$ and it indicates a small time delay τ between ϕ_3 and ϕ_m to show the nonzero feedback. If information bit "1" is used for modulation, it indicates that ϕ_m should make ϕ_3 faster. Their values can be written as $\phi_3(t) = \phi_m(t - \tau)$ where $\tau > 0$. As both ϕ_3 and ϕ_m increase linearly, their difference $\theta_{m-3} > 0$ in most cases. On the contrary, if information bit "0" is encountered, the phase difference is $\theta_{m-3} < 0$ in most cases, as observed in Fig. 4(d).

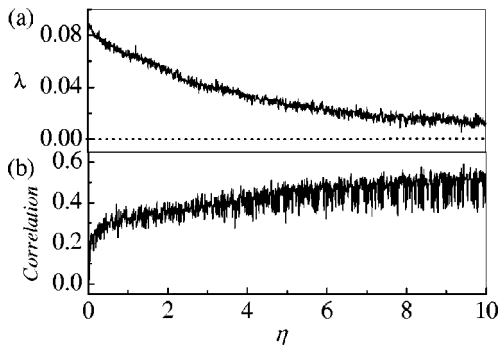The mechanism of our proposed scheme is much differ-

FIG. 5. The evolution of (a) the maximum conditional Lyapunov exponent (CLE), and (b) the maximum cross correlation of amplitudes r_m and r_3 vs the coupling strength at response subsystem.

ent from that of parameter modulation corresponding to CS. In the latter scheme, the nonidentical drive-response system represents bit ''1'' while the identical setup represents ''0.'' At the response subsystem, the information is recovered by calculating the difference between the transmitted and the reconstructed signals. It corresponds to the nonsynchronization or synchronization state between the drive and the response subsystems. However, with our method, the phase is measured according to the small time delay τ and PS is maintained even for two different oscillators.[43] Therefore, the detection of ''1'' or ''0'' in the response subsystem is related to different time delays between the drive and the response phases. If we use some other parameter values in the response subsystem, PS is always maintained, but the time delay that appears as fluctuations below or above zero cannot reflect the correct information bits.

Unlike CS that utilizes state variables as the transmitted signals, our model only uses the phase of the drive subsystem while the amplitude information is not transmitted at all. As a result, the amplitudes between the drive and the response subsystems remain chaotic during the decoding process although the coupling strength in the response subsystem is rather strong. By this means, their maximum CLE remains positive even with a large value of η. This can be found from Fig. 5. In Fig. 5(a), with the increase of η from 0 to 10, the maximum CLE reduces gradually, but still remains positive. Figure 5(b) shows the corresponding maximum cross correlation between the amplitudes r_m and r_3, where r_m is the amplitude of the mean attractor (x_m, y_m). At $\eta = 5.3$, their maximum cross correlation is about 0.45, which is still weak. This property is different from that of CS, where negative CLE of the response subsystem is necessary to obtain the strong coherence of amplitudes between the drive and the response subsystems.[5] As our scheme can decode information from θ_{m-3} with chaotic phenomenon of the amplitude difference, the response subsystem does not necessarily possess negative CLEs.

III. COMMUNICATION USING SIGNALS WITH BOUNDED AMPLITUDE

As the transmitted signal ϕ_m increases linearly, it is impossible to utilize in practical communication channels. Its amplitude fluctuation ξ_m also appears Markov random phe-
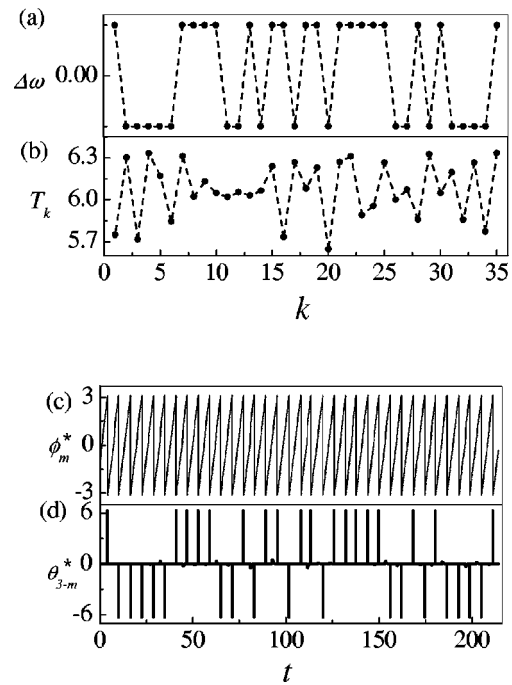

FIG. 6. (a) The message ''ARMY!'' in ASCII format is modulated into the parameter $\Delta\omega$, and (b) the corresponding time period T_k of the transmitting signal ϕ_m^* vs the index k. (c) The portraits of ϕ_m^* transmitted from the drive subsystem to the response one. (d) The phase difference θ_{3-m}^* whose fluctuations represent demodulated information bits.

nomena with unbounded amplitude[48] in infinite time evolution. Moreover, the small fluctuation shown in Fig. 4(d) is always ambiguous and difficult to be detected correctly. By this means, we should use another quantity with bounded amplitude for transmission. To facilitate our analysis, we use a simple way to decode the binary information at the response subsystem.

The problems mentioned in the last paragraph can be overcome if ϕ_m varies in the range $[-\pi, \pi]$ instead of increases linearly. We can make use of Eq. (4) directly to obtain the period of transmitted signal marked as ϕ_m^*. When ϕ_m^* turns from π to $-\pi$, which corresponds to the Poincaré surface in Fig. 2(b), a new information bit is used to perform the modulation in the coupled chaotic oscillators through $\Delta\omega$, as governed by Eq. (1).

In the following, we will give some examples to illustrate how binary sequences are modulated in the chaotic fluctuation. Suppose that we intend to send the message ''ARMY!'' in the following ASCII format:

$$\begin{matrix} A & R & M & Y & ! \end{matrix}$$
100000111100101101101111001010000.1

After PS is obtained between the drive and the response subsystems, set $\eta = 5.3$ and $\Delta\omega$ is modulated with the information bits plotted in Fig. 6(a). With the change of $\Delta\omega$, the two coupled and identical oscillators in Eq. (1) produce an instantaneous mean phase ϕ_m^*, as shown in Fig. 6(c). In this figure, ϕ_m^* seems quite regular and one can easily obtain the values of each period T_k from ϕ_m^*. Indeed, the change of $\Delta\omega$ can affect the rotation period. However, the effect is negligible as the change of period can be expressed as ΔT
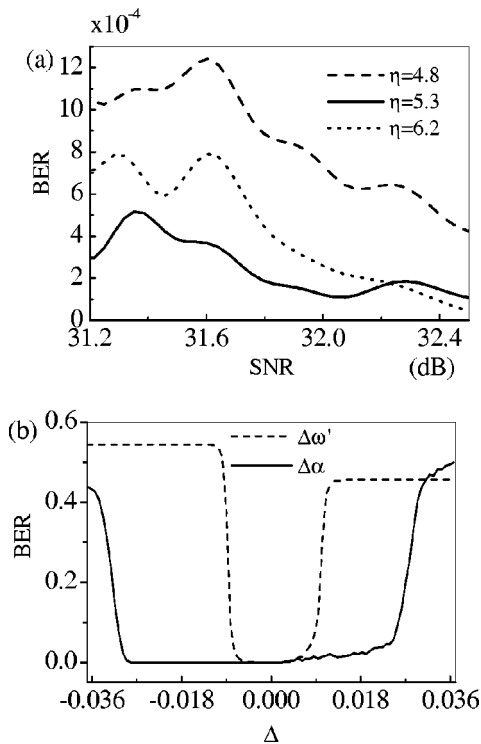
FIG. 7. (a) BER under additive white Gaussian noise in the transmission channel. (b) Relationship between BER and parameter mismatch of the response subsystem.

$= 2\pi/\omega - 2\pi/(\omega \pm 0.01) \approx \pm 0.06$, which is quite small when compared with the chaotic fluctuation of T_k. By this means, their periods T_k still appear chaotic fluctuation, as observed in Fig. 6(b). Evidently, it is impossible to extract the binary information shown in Fig. 6(a) from the fluctuation of T_k. After ϕ_m^* is transmitted to the response subsystem given by Eq. (2), their phase difference $\theta_{3-m}^* = \phi_3^* - \phi_m^*$ is measured and plotted in Fig. 6(d). Comparing Figs. 6(a) and 6(d), if the information bit is "1," θ_{3-m}^* has a positive jump at the transition from one period to the other, i.e., from $+\pi$ to $-\pi$. If the information bit is "0," θ_{3-m}^* has a negative jump instead. Similar to the explanation in Sec. II, these jumps are produced by the small time delay between ϕ_3^* and ϕ_m^*. At a specific time when one phase switches from $+\pi$ to $-\pi$ and the other one is near $+\pi$, the phase difference has nearly 2π jumps, as shown in Fig. 4(d). These fluctuations can be easily detected in the response subsystem. As a result, θ_{3-m}^* has a series of positive or negative spikes, which correspond to the signal "1" or "0."

IV. ANALYSIS ON ROBUSTNESS

The effect of varying the precision of ϕ_m^* on the bit error rate (BER) as a function of the Gaussian random noise was evaluated through simulation and is shown in Fig. 7(a). In each simulation, 1×10^5 randomly generated bits were transmitted. The noise tolerance capability is higher than that reported recently[49] where digital signals are transmitted with cascaded heterogeneous chaotic systems by means of CS. The simulation also shows that there is a value of η that has the largest noise tolerance capacity, which is named optimal

η. At $\eta = 5.3$, the capacity is higher than both $\eta = 4.8$ and 6.2 when SNR < 32.24. However, when SNR > 32.24, the noise tolerance capacity at $\eta = 6.2$ is better.

Here, the optimal η is mainly determined by the decoding scheme and the interaction between the noise and θ_{m-3}. Based on the scheme proposed in Sec. II, information is decoded from the negative or positive values of θ_{m-3}. In the case without noise, although the BER can be reduced by an increase of η, the values of θ_{m-3} also decrease because of strong coupling. In other words, a large η leads to a small θ_{m-3}. If noise is added into the phase value that is transmitted from the drive to the response subsystems, the results can be approximately represented by $\theta_{m-3}^{\text{noise}} = (\phi_m + \text{noise}) - \phi_3$ $\approx \theta_{m-3} + \text{noise}$. Here, we neglect the effect of noise to ϕ_3 because it is rather small when comparing with ϕ_m. At a certain noise level, when θ_{m-3} decreases, there is a higher chance that it has a contrary direction with $\theta_{m-3}^{\text{noise}}$, e.g., $\theta_{m-3} > 0$ but $\theta_{m-3}^{\text{noise}} < 0$. This in turn indicates a higher sensitivity to noise perturbation and so η should not be too large. As a result, there exists an optimal η that leads to a low BER with relatively good noise tolerance capability. The analysis also indicates that the optimal η is not a constant value at a variety of noise density. Thus $\eta = 5.3$ is just an approximate optimal value at SNR < 32.24.

Another typical factor that degrades the communication performance is the nonidentical parameters of the drive and the response subsystems. In practical applications, we cannot make all parameters absolutely identical. One has to consider the case of a slightly different response system by introducing a small parameter mismatch. Figure 7(b) shows the relationship between BER versus the parameter mismatch. For ω', it requires that $|\Delta\omega'| < 0.01$ in order to maintain good performance. This is evidently a result that the modulated parameter $\Delta\omega$ switches between 0.01 and -0.01, as governed by Eq. (6). On the other hand, a much larger mismatch is allowed for the parameter α because it is less sensitive to the change of the system's mean frequency.

V. SECURITY ANALYSIS

By the analysis on the inherent characteristics of CS, many approaches have been put forward to attack the secure communication systems based on this type of synchronization. Among them, the dynamics-based method reported in Refs. 50 and 51 and the parameter identification method described in Refs. 52 and 53 are considered the most effective methods. As PS is different from CS, it is difficult to break our scheme with these methods. (1) *Return map method*: Given one of the variables in the chaotic system, one can properly construct a return map where the dynamics are attracted to an almost one-dimensional set. As the time period of T_k can be easily obtained from ϕ_m^* shown in Fig. 6(c), attacks can use the return map of T_k to extract the information bits. The time period T_k of ϕ_1 (or ϕ_2) has simple return map, as shown in Fig. 8(a). The small change of $\Delta\omega$ splits the attractor into two close parallel branches representing "1" and "0," respectively. We can classify the points according to which branch of the segment they fall in and then uncover the information bits. However, as we use the phase
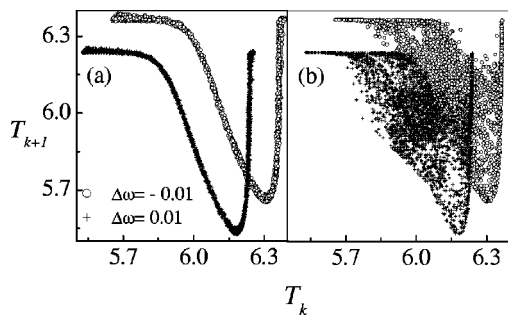
FIG. 8. (a) Return map T_k of one of the identical oscillators in Eq. (1) with only two switches of $\Delta\omega$. (b) Return maps T_k of the mean field.

of the mean field as the transmitted signal, points of the return map do not locate in the vicinity of the pure attractor but are distributed almost uniformly in the large space, as shown in Fig. 8(b). The small change of $\Delta\omega$ cannot split the two attractors clearly. Therefore, it is difficult to extract all the original information from the return map method. (2) *Nonlinear forecasting method*: The chaotic dynamical systems generally exhibit very regular geometric structures that can be used to predict the behavior of the chaotic carrier so that the hidden information can be extracted. It is also required that the transmitted signal is autonomous to ensure a much stronger short-time correlation. In our scheme, we use the phase ϕ_m^* instead of the variables as the drive signal and no regular geometric structure of ϕ_m^* can be found. Moreover, the time duration used to hide information is a discrete variable with chaotic fluctuation, as observed in Fig. 6(b). By this means, the information cannot be extracted correctly. (3) *Parameter identification method*. From the transmission point of view, we usually assume the system architecture is available for the intruders and the system parameters play a role of secrete key in transmission. In Refs. 52 and 53, it is proven that for a drive-response synchronization system, the parameters can be recovered using both the sensitivity of chaotic synchronization to parameter mismatch and optimization algorithm. In our scheme, PS is always robust for various response systems.[43] The information is detected with suitable time delay between the drive phase and the response one. Large mismatch of parameters only leads to large bit error rate of information. For example, in Fig. 3(b), if we put $\omega' > 1.01$ or $\omega' < 0.99$, the BER is close to 0.5. This means the public information is just noise-like although PS is always maintained in the drive-response system. Therefore, one cannot use the sensitivity of chaotic synchronization to search the optimal parameters.

In conclusion, we extend the potential applications of PS to secure communications. Based on the properties of PS, a simple communication scheme is proposed. Information is demodulated from the drive phase with suitable time delay between this signal and the response phases. Simulation results show that this scheme is difficult to be broken by traditional attacks. As the drive-response system is in a PS approach rather than a CS one, a response subsystem with a positive CLE is also allowed.

Further study is necessary to develop schemes that possess a higher security and are more robust to noisy perturba-

tion. For example, instead of using the phase of the mean field as the drive signal, a variable of the mean field can also be used. At the response subsystem, the geometric structure of mean field can be reconstructed by the time delay of the drive variable and then the phase of the mean field is obtained. As the phase of the mean field is measured from only one variable with its time delay, it appears less sensitive to the modulation of parameters of the drive subsystem. By this means, it is necessary to develop a more effective modulation method. Results on such work will be presented in the coming reports.

ACKNOWLEDGMENT

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