



Individual's strategy characterized by local topology conditions in prisoner's dilemma on scale-free networks

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ARTICLE INFO

Article history:

Received 4 October 2008

Received in revised form 23 February 2009

Available online 18 March 2009

PACS:

64.75.+g

61.20.Ja

68.05.-n

Keywords:

Scale-free networks

Prisoner's dilemma

Cooperation

Game theory

ABSTRACT

We introduce a “gradient” to find out the defectors, and further a “topology potential” to characterize the individual's strategy preference in the prisoner's dilemma on scale-free networks. It is shown that the cooperators typically locate on the nodes with high topology potential and the defectors are mainly found on the nodes with small topology potential. A critical topology potential is found for the nodes where cooperators are nip and tuck with defectors. So the information of node's degree, gradient and topology potential together can predict individual's strategy decision in the prisoner's dilemma on the complex networks.

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1. Introduction

Cooperation has played an important role in the major transition of evolution [1,2]. Yet, how the ubiquitous cooperation emerges and evolves in the context of Darwinian evolution is a challenge met by scientists from many different fields of natural and social sciences [3,4]. Evolutionary game theory [5,6] provides a uniform mathematical framework for the problem, and the prisoner's dilemma (PD) [7] used as a metaphor for studying cooperation between independent individuals has been extensively explored. In this game, two players interact with each other through strategies of cooperation and defection. They both get a reward R for mutual cooperation and a punishment P for mutual defection. If one player cooperates while the other defects, their game payoffs are S (sucker's payoff) and T (temptation), respectively. Strategists accumulate their payoffs through playing with their own neighbors and the payoff to a player is in terms of the effect on its fitness. In the prisoner's dilemma game, the ordering of the four payoffs is $T > R > P > S$, which immediately follows that the best strategy for a player is to defect regardless of its counterpart's strategy. But $2R > T + S$ indicates that mutual cooperation will get higher cumulative income. Thus, there is a dilemma. In homogeneous and infinite populations, the individual's selfish and non-cooperative behavior with short-term benefit will prevail under replicate dynamics [1,5,6] without any specific mechanism.

Five specific mechanisms [8,9] for the evolution of cooperation: direct reciprocity, indirect reciprocity, kin selection, group selection, and network reciprocity, have been developed to do away with the dilemma in the standard framework of evolutionary dynamics such as the replicate equation or stochastic process of game dynamics in finite populations. Besides, teaching capability [10], noise [11–13], and social diversity [14], preferential selection [15], asymmetry of connections [16], heterogeneous influences [17] can also be considered as the potent promoters for cooperation. In spatial reciprocity [8,9,18],

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cooperators can form clusters to resist invasion by defectors whose advantage T/R is within a limited range. Generally, the evolution of cooperation on social networks has been discussed in detail and a simple rule for the cooperation to evolve has been concluded [19]. Moreover, the interplay between cooperation evolution and network evolution has also been studied widely [20–25].

However, most of the previous works have focused on the population dynamics on the aggregate level, where the state variables of the system, such as the relative strategy abundances, are averaged over the population. On the other hand, behavioral rules control the system on agent level where the strategy decisions of the individuals are frequently asynchronous and may contain stochastic elements. It is well known that scale-free (SF) networks, widespread in man made and natural systems [26], have been found as a dominating trait for cooperation [27–30]. Specially, cooperation in the BA networks [31] can be enhanced further. The BA network is a kind of heterogeneous network with scale-free degree distribution and a nearly zero clustering coefficient (equal to 0.0056) [32]. Moreover, their age-correlations induce inter-connections of highly connected individuals (hubs) who will be stable cooperators and form a cluster to protect each other from defectors' invasion. The microscopic behaviors of cooperation evolution on different complex topologies have also been discussed in detail by J. Gómez-Gardeñes et al. [33,34]. There they have focused on the dynamics of pure strategists. In their model pure strategists always keep their strategies after the system reaches a stable state. It is reported that the different topology structures lead to different dynamics of structural organizations of cooperator cores and defector cores (consisting of pure cooperators and pure defectors, respectively). More importantly, the inter-connected hubs always form a single cluster of pure cooperators, and result in cooperation enhancement in BA networks. Recently, much attention has been paid to the dynamical features of the game on SF networks [35]. A comprehensive review in this field has been given in Ref. [36].

At agent level, agents may have very different individual preferences of strategy options due to different local environments. So it is important to find out the possible relationship between individuals' strategy preference and its topological condition when the system reaches the stationary state. For each individual, here, the strategy preference is represented by its cooperation frequency at the steady state. It has been shown that the connecting properties, such as degree, especially for the hubs, largely affect agents' strategy preferences on SF networks [29]. However, an important question is if we can find a quantitative topology parameter to describe the individual's strategy preference for each node in a given network. In this paper we will analyze the effects of local topological structure on the individual's strategy preference in the BA networks. Our starting point is to find out defectors in the BA networks by introducing "gradient" for each node and then to characterize one's strategy preference by another quantity "topology potential". A high topology potential for a node indicates a high preference for its individual to be a cooperator. At those nodes with a critical topology potential, cooperators are nip and tuck with defectors.

2. Numerical simulation model

Firstly we perform our numerical simulations of the strategy evolution on the BA network generated via the mechanisms of growth and preferential attachment [31]. Once the network is constructed, the evolution is carried out following the convention [18] by choosing the prisoner's dilemma payoffs as $R = 1, P = S = 0$, and $T = b$, which are all divided by reward. Our simulation is performed with the temptation $b = 2.0$, which is the critical parameter for the proper payoff ranking. The finite population analogue of replicate dynamics is implemented. Initially, cooperators and defectors are distributed randomly in the network with equal probability. In each generation, each individual i in the network plays with its k_i neighbors and accumulates payoff, which is stored as P_i . Then, all the individuals update their strategies synchronously by comparing each one's payoff with a randomly chosen neighbor j . Whenever $P_j > P_i$, the player i adopts the strategy of the player j in next generation with a probability $p = (P_j - P_i) / (\max(k_j, k_i) * b)$. We have verified that asynchronous updating gave us similar results (data not shown here).

Simulations are carried out for a population of $N = 5000$ individuals with $k = 4$. The individuals interact with each other along the network edges. The dynamics of cooperation evolution are implemented as described in Refs. [33,34]. In order to make the system reach a stationary state, we first let the system evolve for 30,000 generations and then a time window of 10^4 additional generations are used to check if the system reaches a stationary state. At time t , the cooperator fraction in the population is termed as $p_c(t)$. The condition for being a stationary state is that the relative fluctuation of cooperator

fraction $F = \sqrt{p_c(t)^2 - \overline{p_c(t)^2}} / \overline{p_c(t)}$ during the time window is smaller than 0.01. Note that we give up the case which has not yet reached the stationary state and restart it. Here, we also define cooperation frequency for each individual as C , which is cooperation times divided by 10^4 . Once a stationary state is reached, we calculate the cooperation frequency C for each individual on the network. Each individual's cooperation frequency is averaged upon 100 different initial conditions for each realization of BA network, and then we perform 100 different realizations of the BA networks to analyze our results.

3. Results

To study the relationship between individual's strategy preference and its topological condition, we need to find out some topology quantities. Here we introduce two new topology quantities based on the cooperation-promoting mechanism of SF networks.

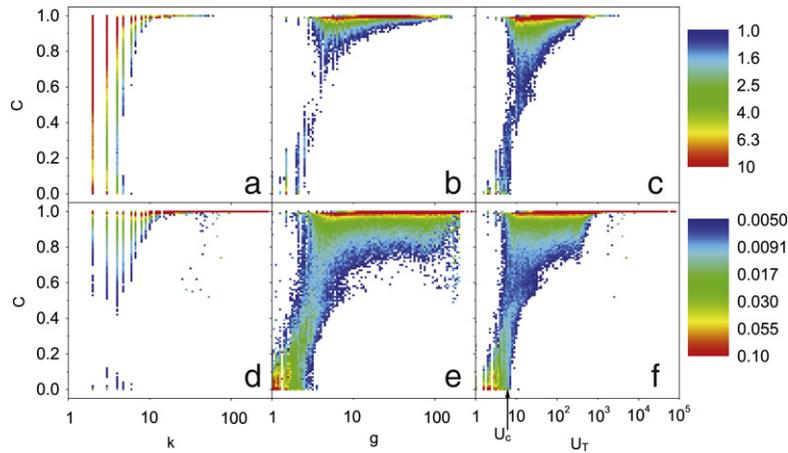


Fig. 1. (color online). (a), (b), (c) The distribution of cooperation frequency for all the nodes with respect to their (a) degree, (b) gradient, and (c) topology potential. The colored points represent different node numbers. (d), (e), and (f) are the results normalized by node number of each abscissa's value. The colored points describe the percentage distribution. Semilog coordinates are used to display the whole range of every plot. (f) The arrow shows the critical value of topology potential, where cooperators are nip and tuck with defectors. All the simulation results are performed using temptation $b = 2.0$. There is no qualitative difference for other values of b .

3.1. The role of node's gradient

The cooperation-promoting feedback mechanism of SF networks lies in the high connectedness of the hubs [27–30]. In SF networks, individuals accumulate different payoffs throughout one generation. All hubs can win over the neighbors because of their large cumulative payoffs. Therefore, the neighbors of a hub will imitate its strategy, which eventually results in a cloud of homogeneous strategists around each hub. Such a process strengthens the cooperative hubs but weakens the defective hubs. As a result, hubs are occupied by cooperators whereas defectors manage to survive only at moderate degree nodes, which is verified in Fig. 1(a). However, it is important to study the strategy options of individuals with small degree because they are main parts in BA networks in virtue of the scale-free degree distribution (Fig. 2(a)). Our first step is to seek out a new topological quantity to describe the defective individuals.

Our simulations record down each node's cooperation frequency at the stationary state of the system. Then we show all nodes on the plane of cooperation frequency and node degree. As a result, the distribution of node number on the plane of cooperation frequency and node's degree is plotted in Fig. 1(a). It shows apparently that the strategy preferences of small degree individuals have widely distributed cooperation frequency. Thus the degree is not competent for describing the strategy preferences of small degree individuals. For a small degree individual, its strategy is unstable and easy to be influenced by its neighbors because of its low cumulative payoff. When such an individual connects to a hub, it will imitate the hub's strategy to be a cooperator. On the other hand, if it is surrounded by small degree individuals, defection will be its stable strategy as the case in homogeneous networks [14,37,38].

In fact, for a certain node, all its connected neighbors will influence its strategy preference. Let us consider an edge connecting a large and a small degree node, which carries a large degree difference. The large degree node will be more stable to be a cooperator just as hubs. Such an edge will lead the small degree node to be a cooperator with large probability. Since cooperators benefit from the edges with large absolute degree differences to a certain extent, we introduce a "gradient" g_i for node i , defined as the average Euclidian distance of degrees with its neighbors:

$$g_i = \sqrt{\frac{1}{k_i} \sum_{j \in v_i} (k_i - k_j)^2}, \quad (1)$$

where v_i is the set of the i th node's neighbors, and k_i is its corresponding degree.

The distribution of node number on the plane of cooperation frequency and gradient is plotted in Fig. 1(b). One can find that the gradient is a better measurement in comparison with the node's degree (Fig. 1(a)) to describe the individual's strategy preference on small degree nodes. The nodes with small degree can be occupied by cooperators as well as defectors. However, the nodes with small gradients are more inclined to be occupied by defectors, while the nodes with large gradient will be occupied by cooperators mostly as shown in Fig. 1(b). Thus the node's gradient can help us to find out defectors. In general, the cooperation frequency of individual arises along with the node's gradient. At larger temptation b , this trend becomes more obvious (data not shown here). However, when $b < 2$, the cooperators dominate completely and it is a little difficult to differentiate the defectors and cooperators.

In order to further study the different roles of node's degree and gradient, we plot the node number distribution on the plane of gradient and degree in Fig. 3(a). One can find that the nodes with large degree will have large gradient. Due to the scale-free degree distribution (Fig. 2(a)), hubs are surrounded by small degree nodes so that they also have large gradient.

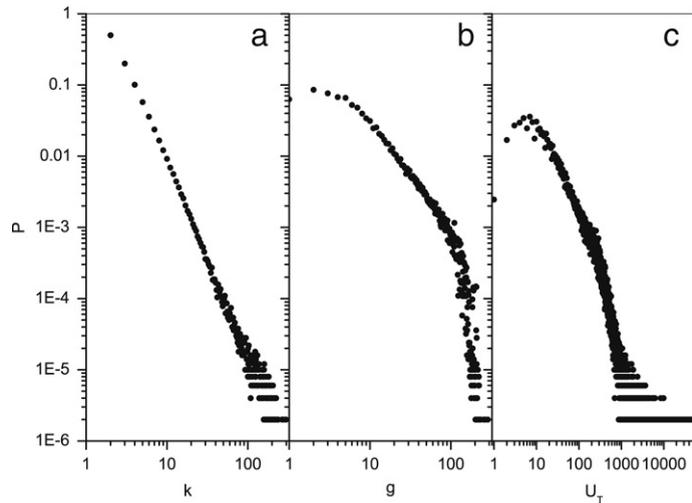


Fig. 2. (a) Scale-free distribution of degree; (b), (c) Multi-scale distribution of gradient and topology potential, respectively.

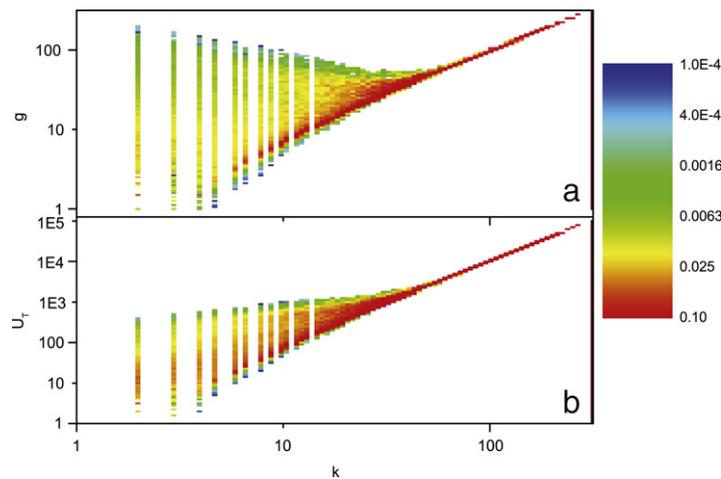


Fig. 3. (color online). The percentage distribution on the plane of degree and gradient (a) and of degree and topology potential (b) with double logarithmic coordinates. The colored points represent the percentage distribution for each degree value.

On the other hand, in the small degree region, node's gradient is not associated with its degree. These nodes have a wide gradient distribution, because their gradient are mainly determined by their neighbors. Most small degree nodes link to the few hubs and their gradients are also large, almost equal to the hubs' gradient as shown in Fig. 3(a). Individuals on these nodes will frequently behave as cooperators by imitating hub's strategists. But the rest nodes with small degree are linked to homological neighbors and their gradients are small. These nodes will mainly be occupied by defectors. So this kind of connection differentiates the small degree nodes with different cooperation frequencies through the evaluation of their gradients.

3.2. The role of node's topology potential

In Fig. 2(b), we plot the distribution of gradient, which is multi-scale but not even ranging from 1 to 100. Due to the uneven distributions of degree and gradient, the node number distribution on the planes between cooperation frequency and topology quantities is not clear as a whole although they can show us some relationships between cooperation frequency and topology conditions as discussed above. In fact, at a given value of topology quantity, the strategy preferences of individuals can be characterized by individuals' cooperation frequency distribution. So we plot the node's cooperation frequency distribution, normalized by node number at each abscissa's value, with respect to degree and gradient as shown in Fig. 1(d), and (e), respectively. One can find that degree is competent for describing the cooperators for large degree nodes, but bad otherwise. On the other hand, gradient is proper for describing the defectors for small gradient nodes, but still not so good for large gradient nodes. For the nodes with large gradient but small degree, they will get low payoffs from the interactions with their neighbors and imitate strategies of the large degree neighbors that often behave as cooperators. But

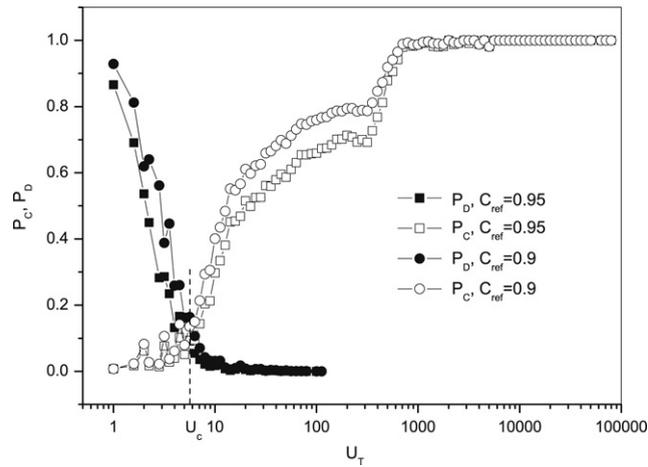


Fig. 4. The fraction of various pure cooperators P_C and pure defectors P_D as a function of topology potential with different reference C_{ref} . At a critical value U_c of topology potential, the fractions of pure cooperators and pure defectors get cross to each other for both references of cooperation frequency C_{ref} . In the case with $b = 2.0$, the critical value U_c equals 5.6.

they will frequently be invaded by their small degree defective neighbors, too. So the large gradient nodes have a relatively wide distribution of cooperation frequency.

As a result, the degree and gradient can be actually competent for describing the strategists on large degree nodes and small gradient nodes, respectively. So we take a further step to introduce another quantity “topology potential” U_T to describe the individual’s strategy preference by using a simple coupling between node’s degree and gradient:

$$U_T = k \cdot g. \quad (2)$$

Its distribution is multi-scale, ranging from 1 to 10^5 , as shown in Fig. 2(c).

The node number distribution and its normalized distribution on the plane of cooperation frequency and topology potential are plotted in Fig. 1(c) and (f), respectively. We can find that the strategy preference of individuals can be best described by topology potential compared to the former two quantities. The nodes with small topology potential will mostly be occupied by defectors, while the cooperators will take up the nodes with large topology potential. With an increase of the topology potential, more and more cooperators will occupy these nodes, leading to an increase of the fraction of cooperators (Fig. 1(f)).

The relationship between topology potential and degree is also important for discussing the topology potential’s function on determining individual’s strategy preference. Fig. 3(b) shows the node number distribution on the plane of topology potential and degree. It tells us that the topology potential also differentiates the nodes with small degree as gradient does, and the large degree nodes have large topology potential. Note that the nodes with small degree have relative small topology potential as compared to the nodes with large degree attributing to the coupling between degree and gradient. This is different from the case of gradient (Fig. 3(a)). Individuals on small degree nodes are frail, and their strategies are determined by their neighbors. Sometimes they may imitate cooperative neighbors to be cooperators, or they can be defectors by being invaded by defective neighbors. So individuals on these nodes may spend a certain fractional time for cooperation, which is well characterized by their moderate topology potentials.

What is more, the topology potential can differentiate pure cooperators and pure defectors. Here, we introduce a definition of pure strategists different from that in Refs. [33,34]. A player is defined as a pure cooperator if its cooperation frequency is larger than a given reference C_{ref} , or a pure defector if its cooperation frequency is smaller than $1 - C_{ref}$. The value of C_{ref} ranges from 0.5 to 1. Our definition is the same as that in Refs. [33,34] when $C_{ref} = 1$. With a given C_{ref} , the population can be partitioned into three kinds of strategists: pure cooperators, pure defectors and intermediate strategists. Furthermore, among the nodes at a given topology potential U_T , we introduce the fractions of pure cooperators $P_C(C_{ref}, U_T)$ and pure defectors $P_D(C_{ref}, U_T)$. The fractions of pure cooperators P_C and pure defectors P_D as a function of topology potential U_T with different reference C_{ref} are plotted in Fig. 4. It shows that most individuals are pure defectors at nodes with small topology potential. With increasing the topology potential, the fraction of pure defectors decreases rapidly, while that of pure cooperators increases.

At a certain value of topology potential, the fractions of pure cooperators and pure defectors get cross each other. We refer it as the critical U_c . The simulations show that the critical value U_c of topology potential almost keeps fixed with varying C_{ref} (Fig. 4). In the case with $b = 2.0$, the critical value U_c equals to 5.6. By comparing the value of topology potential of an individual with such a critical value, one can roughly anticipate the cooperative probability of its partner. When one’s topology potential is larger than the critical value, the individual will behave as a cooperator with large probability because there are more pure cooperators than pure defectors for any C_{ref} among the population with the same topology potential. Conversely, if one’s topology potential is smaller than the critical value, the individual will probably be a defector.

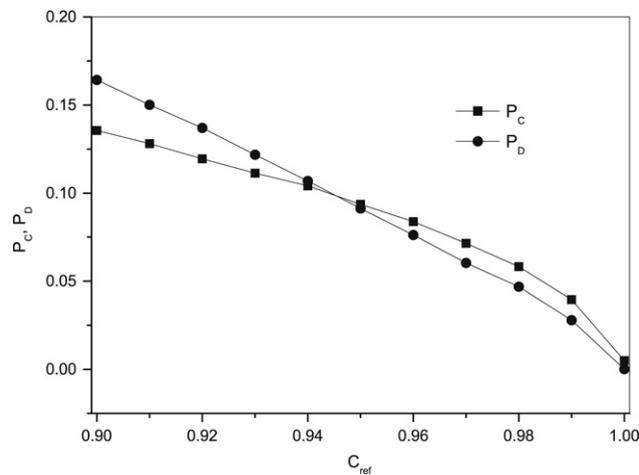


Fig. 5. The pure cooperator fraction P_C and pure defector fraction P_D as a function of C_{ref} at the critical value U_c of topology potential.

So the information of topology potential can predict one's strategy decision. However, there are also difficulties in the case when one's topology potential is close to the critical value. Both fractions of pure cooperators P_C and pure defectors P_D decrease with C_{ref} almost linearly on the nodes with critical topology potential (Fig. 5), which indicates that the cooperation frequency of individuals on these nodes is widely and symmetrically distributed. At the critical value of topology potential, the defectors' advantage is cancelled out by the network's heterogeneity and cooperators are nip and tuck with defectors (Fig. 1(f)).

4. Summary

In this paper, the relationship between individual's strategy preference and its topological features is discussed in PD game on scale-free networks. Besides the node's degree, we introduce two more quantities, i.e. gradient and topology potential, as individual's features. Here, the gradient is defined as the average Euclidian distance between a node's degree and the degrees of its neighbors, while the topology potential is a simple coupling between the node's degree and gradient. We show that individuals on large degree nodes are usually cooperators. The small gradient nodes, which typically have a small degree and are also linked to similar small degree nodes, will normally be occupied by defectors. The nodes with large topology potential are typically either nodes with a large degree or nodes with a small degree but linked to a large degree node. The nodes with small topology potential are the nodes with small gradient. We show that the cooperators usually locate on the nodes with high topology potential and the defectors are mainly found on the nodes with small topology potential.

We suggest that the three quantities (i.e. degree, gradient and topology potential) together can discriminate nodes in BA networks. Based on these features, individual's cooperation frequency can be estimated. We furthermore suggest that the three quantities may play an important role in studying coevolution of strategy and structure in other complex networks.

Acknowledgments

This work is supported by the National Science Foundation of China under Grant No. 10225420 and J. W. Shuai acknowledges support from the National Science Foundation of China under grant No. 10775114.

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